

# How Can We Quantify the Current State of Multi-Decadal Phenomena Like the PDO/AMO

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# Choose Indices That Maximize Your Interests

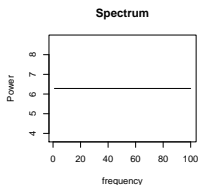
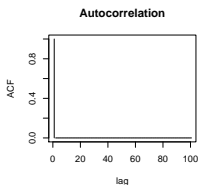
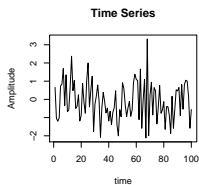
## **Decadal Variability**

Maximize Low-Frequency Power (EOFs of Low-Pass Filtered Data)

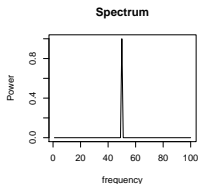
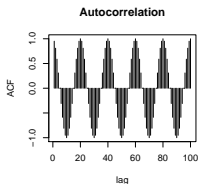
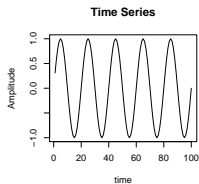
## **Decadal Predictability**

Maximize the **Ratio** of Low-Frequency Power to Total Power

# Relation Between Power Spectrum, ACF, and Predictability



White Noise  
(Unpredictable)



Sinusoid  
(Predictable)

# What Is a Measure of Decadal Variability?

Predictability is related to the width of spectral peaks:

- ▶ Narrow peaks → highly predictable
- ▶ Wide peaks → unpredictable

Measure of “peakiness” at decadal time scales:

$$\frac{p_{10}}{\int p_{\omega} d\omega},$$

where  $p_{\omega}$  is the power spectrum.

# Relation Between APT and Power Spectrum

- ▶ A general measure of “peakiness,” regardless of location, is

$$APT = 2\pi \int_{-\pi}^{\pi} \|p_{\omega}\|^2 d\omega$$

where  $p_{\omega}$  is the normalized power spectrum.

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- ▶ The minimum APT occurs for  $p_{\omega} = 1/(2\pi)$  (white noise).
- ▶ APT is a measure of “peakiness” of the power spectrum.

$$\overline{p^2} = \overline{(p - \bar{p})^2} + \bar{p}^2$$

# Optimize APT

Find projection of the data that maximizes APT. Solution:

$$2 \int_0^{\infty} (\mathbf{C}_{\tau} \mathbf{C}_0^{-1} \mathbf{C}_{\tau}^T d\tau) \mathbf{q} = \lambda \mathbf{C}_0 \mathbf{q}$$

where  $\mathbf{C}_{\tau}$  is the time-lagged covariance matrix.

- ▶ Eigenvalue  $\lambda$  gives the APT.
- ▶ Eigenvectors  $\mathbf{q}$  are projection vectors for generating time series.
- ▶ Resulting time series are uncorrelated in time.
- ▶ Each projection vector is associated with physical pattern  $\mathbf{p} = \mathbf{\Sigma}_c \mathbf{q}$ .
- ▶ Physical pattern  $\mathbf{p}$  is called a **predictable component**.
- ▶ Product of  $\mathbf{p} * (\text{time series})$ , summed over all components, recovers original time series.

# Application to Meteorological Data

- ▶ NCEP/NCAR Reanalysis
- ▶ 1000hPa Zonal velocity (U1000)
- ▶ 50-years (1956-2005)
- ▶ 6-hourly data (73,052 time steps)
- ▶  $2.5^\circ \times 2.5^\circ$  grid (10368-dimensional state space)
- ▶ Truncate to the first 50 PCs of standardized U1000

**Forecast Model:** Linear regression model

**Lead times:** 1-180 days.

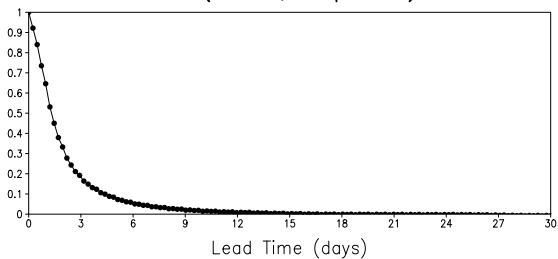
**Training:** Model trained using 1956-1980 data.

**Verification:** Forecast error variance computed from 1981-2005.

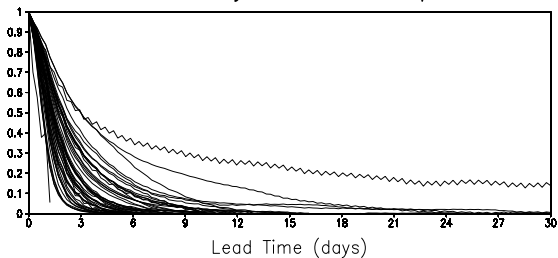
**Statistical Significance:** 5% significance relative to no-skill model.

# Total Predictability of U1000

Predictability of Global U1000  
(50EOFs, Independent)

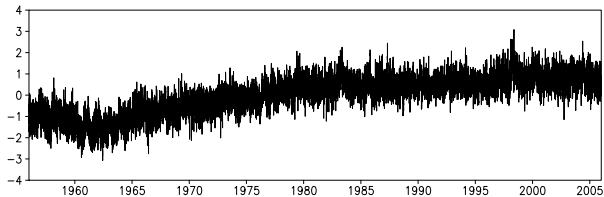
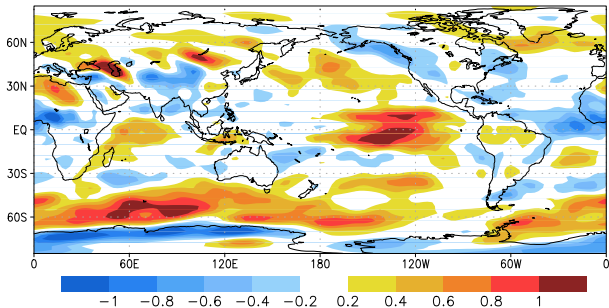


Predictability of Individual Components



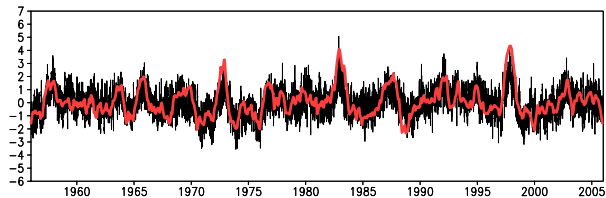
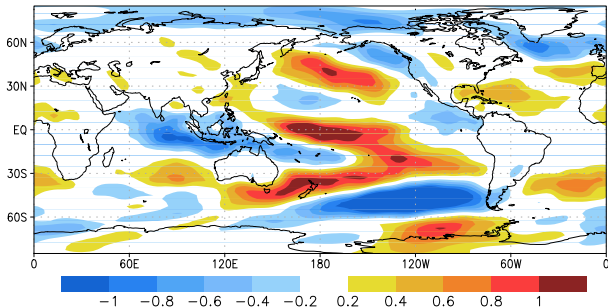
# First Predictable Component

Predictable Component 1  
(U1000, APT=84days, 1.4%, 50EOFs)



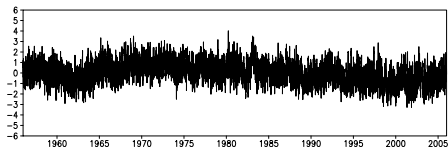
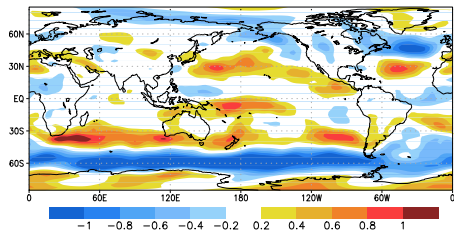
# Second Predictable Component

Predictable Component 2  
(U1000, APT=34days, 1.1%, 50EOFs)



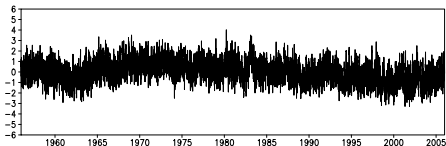
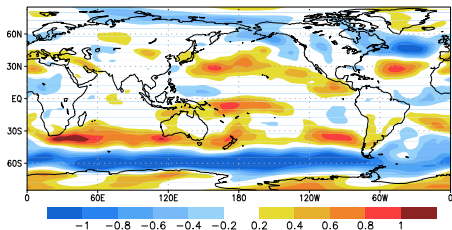
# Third Predictable Component

Predictable Component 3  
(U1000, APT=21 days, 0.8%, 50EOFs)

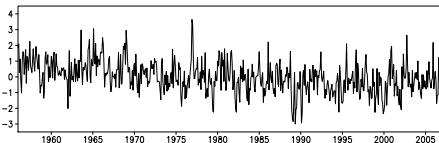
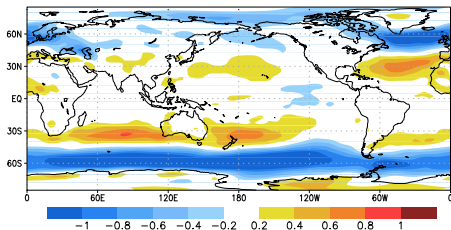


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Predictable Component 3  
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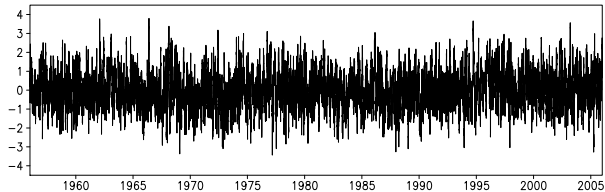
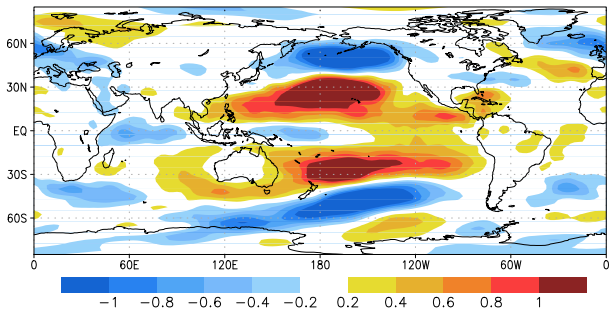


Regression Between AO+AAO and U1000  
(Monthly Mean, 1956–2006)



# Fourth Predictable Component

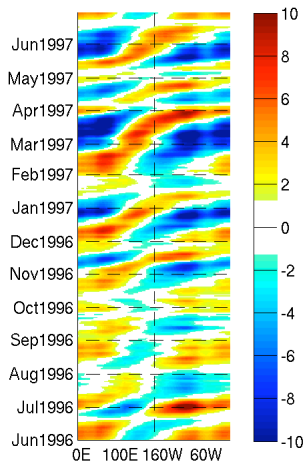
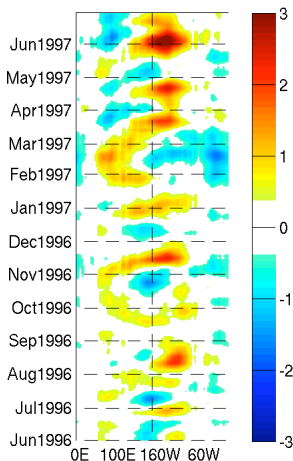
Predictable Component 4  
(U1000, APT=15days, 1.2%, 50EOFs)



# Higher Predictable Components (MJO?)

PrC Reconstructed  
(DelSole and Tippett 2008)

vs. RMM Method  
(Wheeler and Hendon 2004)



**Predictability on climate, seasonal, intraseasonal, and weather time scales was detected in 6-hourly data without time averaging.**

# Proposal: Define Indices Based on Maximizing APT

## Pros

- ▶ No other index can have stronger spectral peaks.
- ▶ No other index can have more linear predictability.
- ▶ Variables with different units can be combined.
- ▶ Indices can be monitored on much smaller time scales than the period of predictability (there's no time averaging window).

## Cons

- ▶ Overfitting! (How many EOFs Should Be Used?)
- ▶ Captures only linear predictability.